STUDIES ON THE EFFECTS OF MAGNETISM on the SCATTERED RESPONSES of CONDUCTORS to INDUCTIVE SOURCES

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This analysis is a simple study of the scattered EM fields created by fields emitted by inductive sources upon conductors which are also magnetic. An inductive source may be a ungrounded loop of wire or a more complex coil antenna. Of course, grounded sources such as long wires carrying current also propagate magnetic fields which will respond differently to magnetic conductors than non-magnetic conductors. Here, we only consider the so-called inductive source and the analyzes is not intended to be comprehensive. For this analyzes, we take the fields to be in the frequency domain and thus can be considered sums of sinusoidal functions.

There are three simple and straightforward effects of conductors being also magnetic. Two can be considered static effects while the third is a straightforward effect to time varying magnetic source fields.

1. **Magnetostatic Effect** - Gauss' Law – the response of a magnetic field on a magnetic body
2. **Galvanic Effect** – Continuity Equation – the effect of currents incident upon a magnetic conductor
3. **Inductive Effect** – Faraday's Law – the inductive response of a magnetic body which is conductive

We will layout the basic theory and follow with some examples. The simulations are primarily computed using extensions to Debye's spherical solution for the EM response of a sphere of uniform conductivity, permeability and permittivity embedded in a similarly uniform background from a magnetic or current source dipole. The solution is comprehensive and precise and incorporates all scattering phenomena. The problem of determining a method to obtain a convergent series using 200 terms was found after numerous attempts at using only a few first terms in the spherical expansion. The method has been extended to the use of inductive loops , ground dipoles and moving systems.

The Magnetostatic Effect: This effect is concisely expressed by Gauss's Law. The scattered field has the phase of the incident field.

$$\nabla \cdot \boldsymbol{B} = 0$$

At low frequency, the resulting field is principally in the first quadrant (i.e. REAL) but as the frequency increases the phase will rotate and the phase of the source field rotates. The rate of this rotation depends upon the conductivity of the material hosting the conductor and the distance through the material from the source to the conductor.

The Galvanic Effect: This effect is sometimes referred to as the current channelling effect. One means to describe this effect is via the representation of the electric fields in terms of a scalar potential (φ) and a vector potential (**A**).

$$\boldsymbol{E} = \nabla \cdot \varphi + i\omega \boldsymbol{A}$$

The scalar potential phi (φ) can be thought of as a result of the continuity equation.

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t} \approx 0$$

For our considerations in geophysics, the divergence of the current density is zero as the charges leave the surface too quickly for our systems to sense. When currents induced within the host media impinge on body with variations in its electric properties then charges are produced on the surface which creates a secondary current distribution with the conductor or magnetic conductor which then produce secondary magnetic fields (H_s).

The Induction Effect: This effect is summarized by Faraday's law. A time varying magnetic field when fluxing across a surface induces electric fields. If the material bounded by this surface is conductive then currents are produced and these currents are sources of electric and magnetic fields.

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{A}$$

Therefore, in principal, any magnetic property of the scattering body will effect the inductive response both induced and permanent. However, the only available algorithm to include the magnetic effects on inductions does not provide for permanent magnetization.

Sources of Secondary Magnetic fields: All of these three scattering effects can produce secondary magnetic fields. One way to think about these secondary fields is in terms of secondary currents produced in a structure. We will only consider for simplicity structure with uniform properties.

The secondary magnetic fields ($\mathbf{H_s}$) due to a body of uniform electrical properties in a uniform background is given in terms of variation of conductivity (σ), magnetic susceptibility (χ) and electric permittivity (ε) as :

$$\boldsymbol{H_s} = \boldsymbol{\nabla} \times \boldsymbol{E}_s(r)$$

$$= \boldsymbol{\nabla} \times [\, i\omega\mu_0\delta(\chi\sigma) + \omega^2\mu_0\delta(\chi\varepsilon)] \int dr \,\{\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}')\}$$

$$= Q \int \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J_s}\, \boldsymbol{dr}'$$

where the secondary currents inside the body can be expressed for a uniform body as:

$$\boldsymbol{J}_s(r') = [\, i\omega\mu_0\delta(\chi\sigma) + \omega^2\mu_0\delta(\chi\varepsilon)]\, \boldsymbol{E}(r') = Q_b \boldsymbol{E}(r')$$

For non-uniform bodies and non-uniform backgrounds, these principles are expanded with various numerical techniques.

Thus, in principle, all variations in material properties effect the amplitude and phase of the secondary magnetic fields. The critical issue effecting the importance of the two scattering processes lies in the nature of the Green's function (**G**) which is a dyadic tensor. This equation does not include the magnetostatic effects.

Understanding Intuitively the Electric Green's Function : All of these three scattering effects can produce secondary magnetic fields. One simple intuitive means to think about these secondary fields is in terms of secondary currents produced in a structure.

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$$\boldsymbol{H_s} = \boldsymbol{\nabla} \times \boldsymbol{E_s}(r)$$

where the secondary fields within the scattering body are $Q_b E(r')$ and therefore the electric fields external are given by

$$\boldsymbol{E_s}(r) = Q_b \int dr \left\{ \boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{E}(\boldsymbol{r}')\boldsymbol{dr}' \right\}$$

EQUATION A

This is now the critical part of the story as the nature of the electric Greens' function (G) makes the EM problem more complicated than potential field problem or even the resistivity problem.

$$\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}') = \left\{ \boldsymbol{I} + {}^{1}\!/_{k^2} \nabla\nabla \right\} g(\boldsymbol{r},\boldsymbol{r}')$$

where k is the background wavenumber and

$$g(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

The Greens' tenor **, G**, comes directly from the form of the two potentials for the electric field (φ and **A**) and thus, **G**, consists of two parts , one with a r-1 singularity and the other with a r-3 singularity. Equation A is true for both locations inside the body and outside the body and therefore when r →r' both portions of the Greens' function approach infinity but the part that represents the scalar potential blows up with much more weight (measure) when integrated over its product with the electric fields inside the body .

Understanding Intuitively the Electric Green's Function :

Equation A can be rewritten as

$$E_s(r) = Q_b\left\{\int g(r,r')E(r')\mathrm{d}r' + {}^{1}/_{k^2}\,\nabla\nabla\cdot\int g(r,r')E(r')\right\} \qquad \textbf{EQUATION B}$$

where now the inductive part and the galvanic parts of the equation are explicitly separated. In this form, two issues are exposed:

1. Both the inductive and galvanic responses are affected by susceptibility variations between the anomaly and the background materials. This will show in the numerical simulations to follow

2. The galvanic response is normally dominant and this has ramifications for the type of survey performed. Such things as the resistivity of the background materials and magnetic source field coupling efficiency with the anomaly play important factors in whether the inductive response can be seen outside of the galvanic response.

Ramifications and Associated Difficulties.

This issue also plays a dominant part in the history of EM simulation algorithms in geophysics. As an example, finite difference, finite element and conventional integral equation approaches to solving for the scattering currents either directly or indirectly have mostly been unable to reproduce the inductive responses in forward modeling solutions. This is caused by formulating the solutions without consideration for the dominance of the galvanic portion of the Greens' tensor and thus the inductive part of the solution is lost in the numerical computations. R.Groom approached this problem by ignoring the inductive portion and solving directly only for the galvanic portion by use of a quai-analytic operator. This approach reproduced with improved accuracy and considerable reduction in computation times that the standard numerical approaches. The one exception to this was a series of formulations for a conductive thin sheet which included J.Lajois and then P.Walker using a type of renormalization. The Walker approach was implemented and slightly augmented by Eikon Technologies.

The problem of having a fast, accurate and effective conductive response has long plagued mining exploration, in particular. This lead to several attempts to produce solutions directly for the magnetic fields omitting the galvanic term. Most of these attempts were of the thin sheet approach where a rectangular prism with no thickness (plate) but an integrated conductivity was used. The first successful solution to this approach was by the Lamontagne company which used an even simpler approach where the currents on the plate where represented by a simple dipole. This approach was duplicated by the EMIT company. Another approach was by eigenvector solution to the inductive problem formulated by Annan and implemented later by Bloore. Annan's solution was later implemented and improved considerably by Eikon Technologies.

Numerical Examples by Model Simulation:

Algorithms.

Four algorithms will be used for the simulation of magnetic conductors: SPHERE, LN, Freespace Plate and VHPlate. These will used for two purposes, first to show the nature of the effects of magnetization on the scattered fields of conductors and second to show the limitations of each algorithm.

For these first set of examples, only inductive sources will be used. The conductors will have varying conductive and magnetic properties but will all be embedded in relatively low resistivity backgrounds as the main purpose of these examples is to demonstrate the effects of magnetization when currents are present in the background. The solutions will all be in the frequency domain with no normalizations applied. All the survey configurations will be fixed source configurations but the location of the source to the conductive body with be varied to illustrate the effects of coupling of the magnetic field on magnetic conductors.

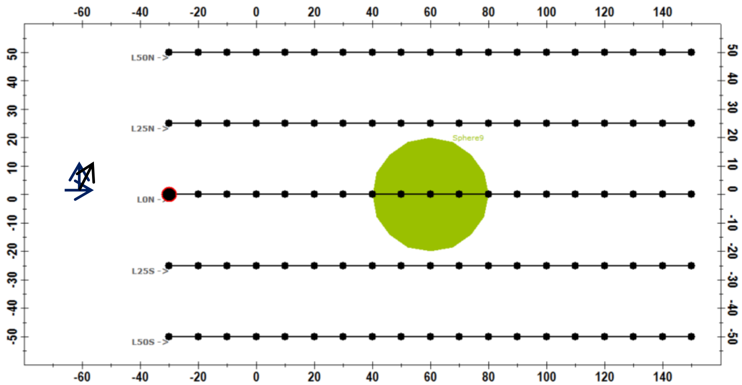
SPHERE: Peter Debye studied EM under Sommerfeld developed the mathematics for the solution for all EM fields in the frequency domain due to a sphere of arbitrary radius and arbitrary variations in conductivity, permeability and permittivity from the whole space in which is was embedded. The solution is for arbitrary electric or magnetic dipole sources at any frequency. The mathematical solution if via spherical harmonic expansion. However, the series is not monotonically convergent and thus without a computer, Debye was not able to find approximations which could be given an error range. In practical terms, one cannot predict for any particular TX and RX location with any set of sphere parameters how many terms are required to reach a given level of accuracy for the solution. J.R. Wait attempted some decades ago to find a numerical solution for the first few terms for certain ranges of parameters but he assumed the spherical series were a monotonic sequence. R.Groom and T.Habashy discovered a method which could rapidly compute 200 terms in the series and determine if the solution had converged within a precision of 1/100th of a percent.

Eikon Technologies has extended the solution to allow fixed or moving systems, frequency or time domain, arbitrary electric & magnetic dipoles, bipoles or loops with a sphere embedded in a halfspace. These developments required little modifications to Debye's solution but rather a series of numerical techniques. It is these solutions that we use there.

Localized NonLinear Approximation (LN): This approach was first proposed by Habashy, Groom and Spies who provided comparisons to the SPHERE solution to demonstrate is ability to reproduce the galvanic effects as outlined above. Subsequently, Eikon Technologies extended the mathematics to develop numerical solutions for prisms and polyhedra embedded in an arbitrary layered background with variations in (σ,μ,ε) from the background. Its solutions reproduces any accurate finite difference, finite element or integral equation solution but the solution is far more rapid and accurate and provide solutions both near the sources and close to and inside the anomalies. The solution does reproduce the magnetostatic effect but does not even attempt to solve the inductive problem. An extension, ILN, or the inductive LN approach does reproduce weak induction.

Freespace Plate: This is the name that Eikon Technologies gives to its mathematical and numerical solutions to P.Annan's PHD formulation for a rectangle of uniform conductance (plate) in a infinitely resistive background with freespace permeability and permittivity. The solution is in terms of the eigenvectors of the scattering matrix and we have developed numerical solutions for up to 11 eigenvectors thus allowing for a much more complex distribution of currents than provided by a single dipole. The solution does not allow for the plate to be magnetized.

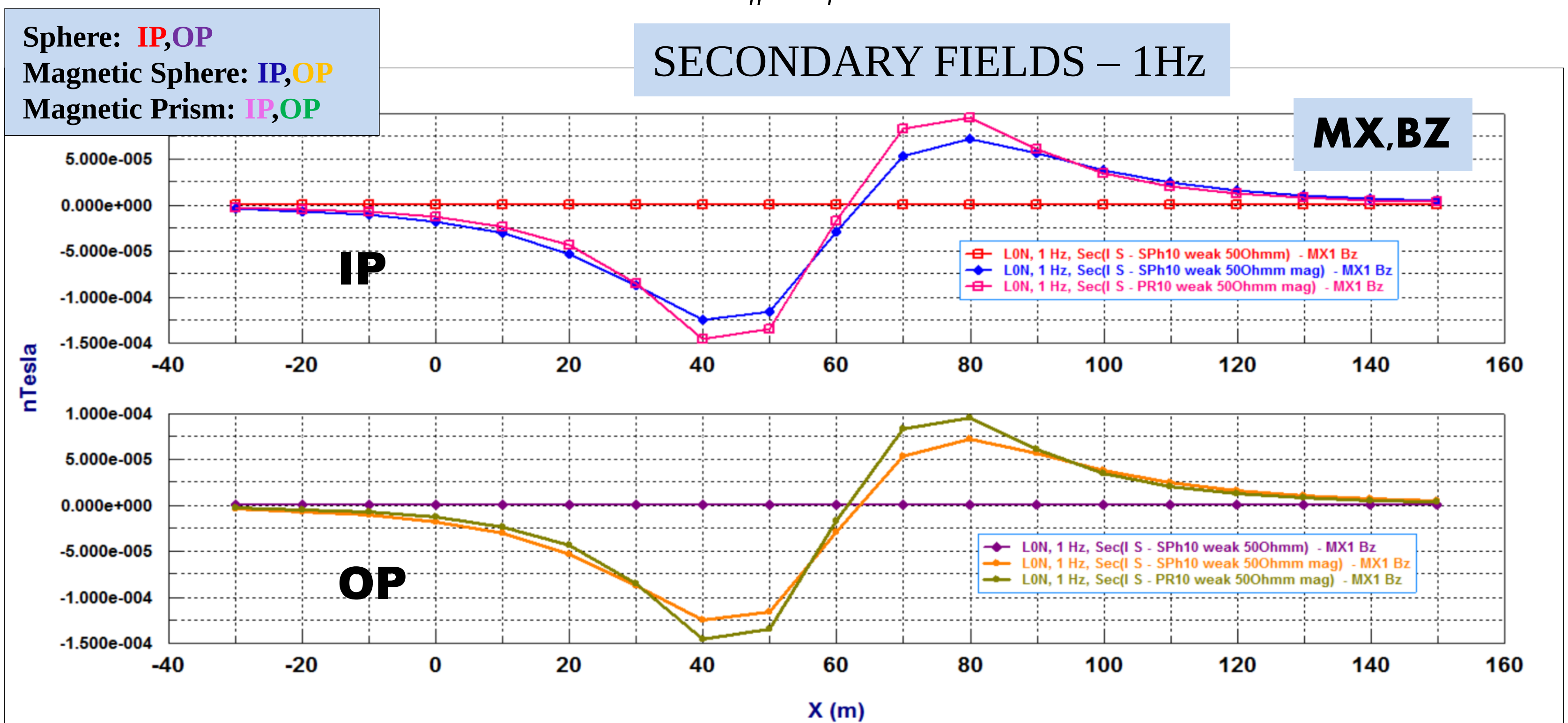
Numerical Examples by Model Simulation : Case 1 Far Transmitters: {weak conductor, magnetic/non-magnetic}



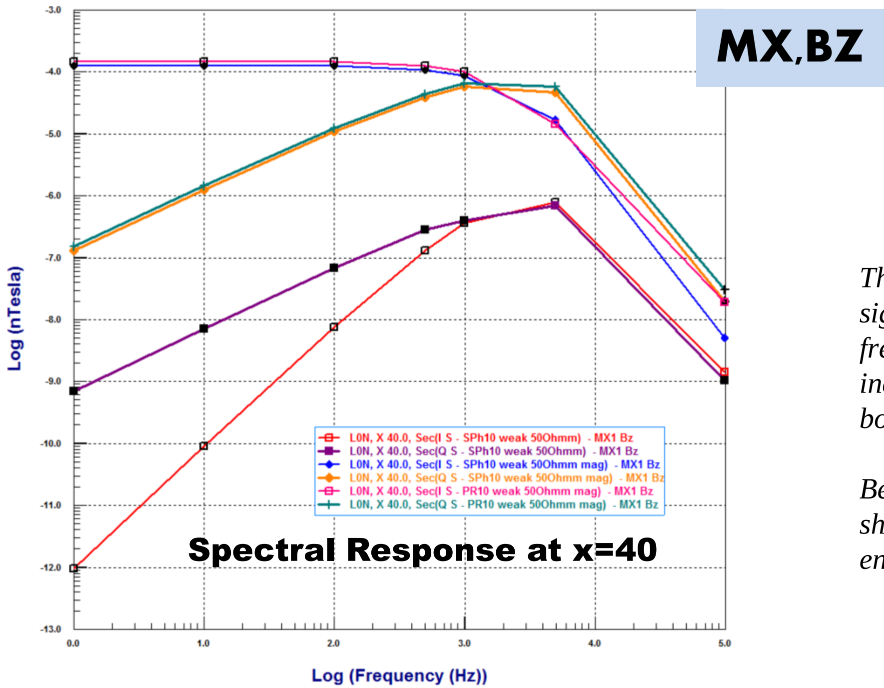
SPHERE: R=20m, (σ=.02,χ=[0,3],ε_r=1), Top=(60,0,-15)
PRISM: 29m cube, (σ=.02,χ=[0,3],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm
3 Dipole Transmitters: (-60,0,0)

This example is for sources small and relatively distant with a quite low background resistivity. The sphere is very weakly conducting w.r.t. the background and is simulated non-magnetically and with magnetization. A prism is also simulated with LN to demonstrate all of the magnetic effects are galvanic. Not only does the magnetization effect the IP but also the OP due to the effects of the induced currents in the host.



Numerical Examples by Model Simulation : Case 1 Far Transmitters: {weak conductor, magnetic/non-magnetic}



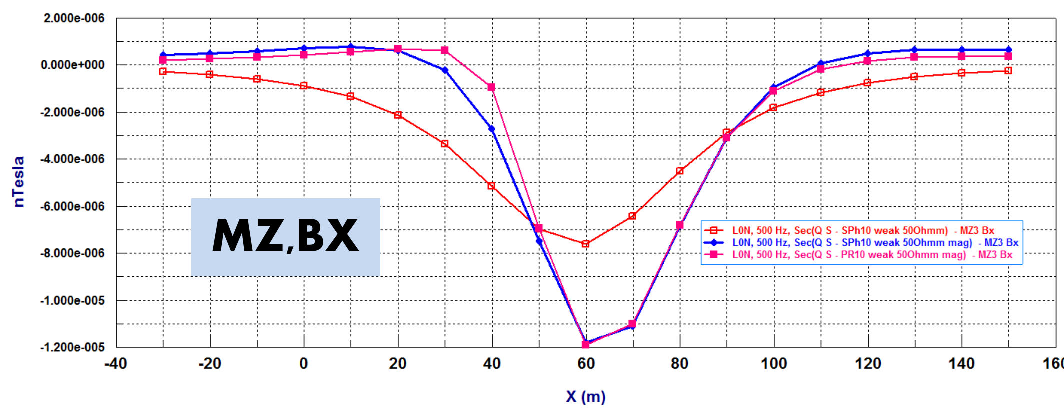
SPHERE: R=20m, (σ=.02,χ=[0,3],ε_r=1), Top=(60,0,-15)
PRISM: 29m cube, (σ=.02,χ=[0,3],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm
3 Dipole Transmitters: (-60,0,0)

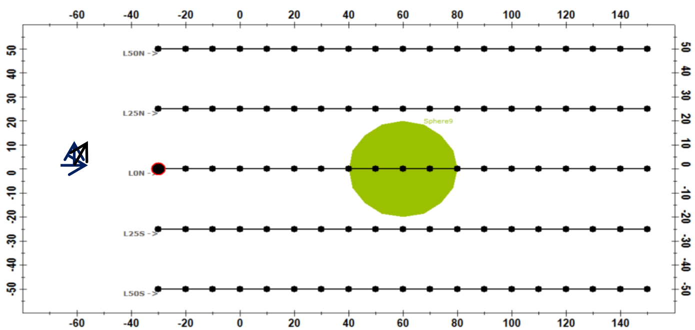
The magnetization causes the IP to be quasi-static at low frequencies but significantly enhances the galvanic effect (current chanelling) in the lower frequencies. At higher frequencies the phase of the background has increased significantly toward 45° but the magnetism has still enhanced both real and imaginary.

Below, the TX is a vertical dipole and the receiver Bx. The quadrature is shown at 500Hz for the sphere and the LN algorithms. The magnetism has enhanced the channelling response but also modified its shape.

Sphere: IP,OP
Magnetic Sphere: IP,OP
Magnetic Prism: IP,OP



Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}

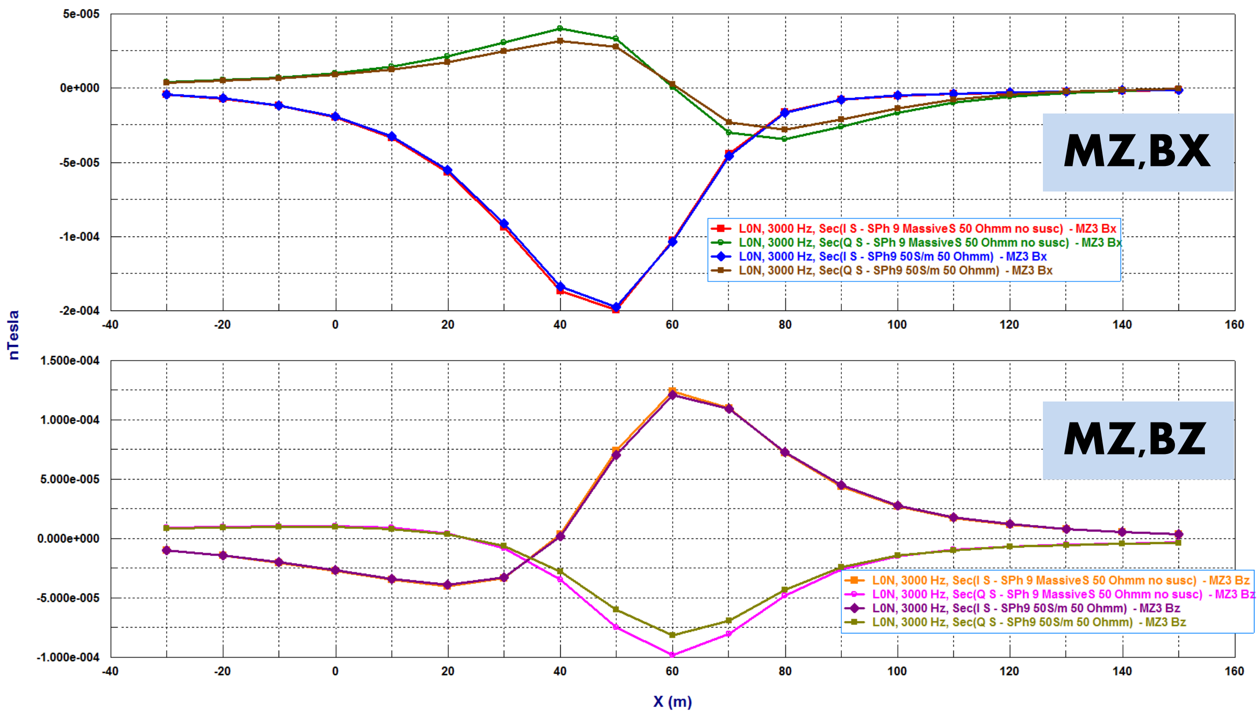


SPHERE: R=20m, (σ=1000,50),χ=[0,10],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm
3 Dipole Transmitters: (-60,0,0)

In this example, the TX's and sphere parameters are as in the previous example but on this page, one sphere is 1000S/m and the other 50S/m. This is to demonstrate that except at lower frequencies, there is an inability to distinguish the conductivity beyond a certain level. Below, there is little to differentiate the EM responses for Bz&Bx and 3KHZ from an inductive source with a vertical moment.

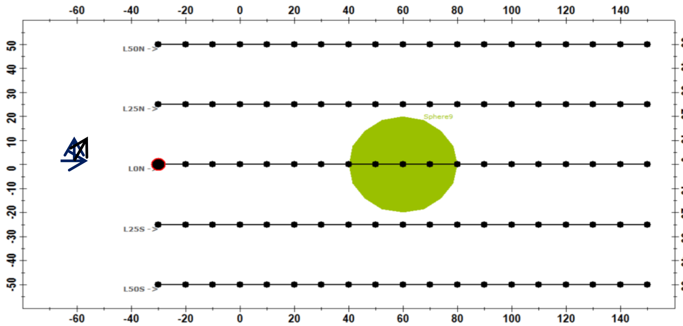
SECONDARY FIELDS



1000S/m Sphere: IP,OP
50S/m Sphere: IP,OP

1000S/m Sphere: IP,OP
50S/m Sphere: IP,OP

Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}

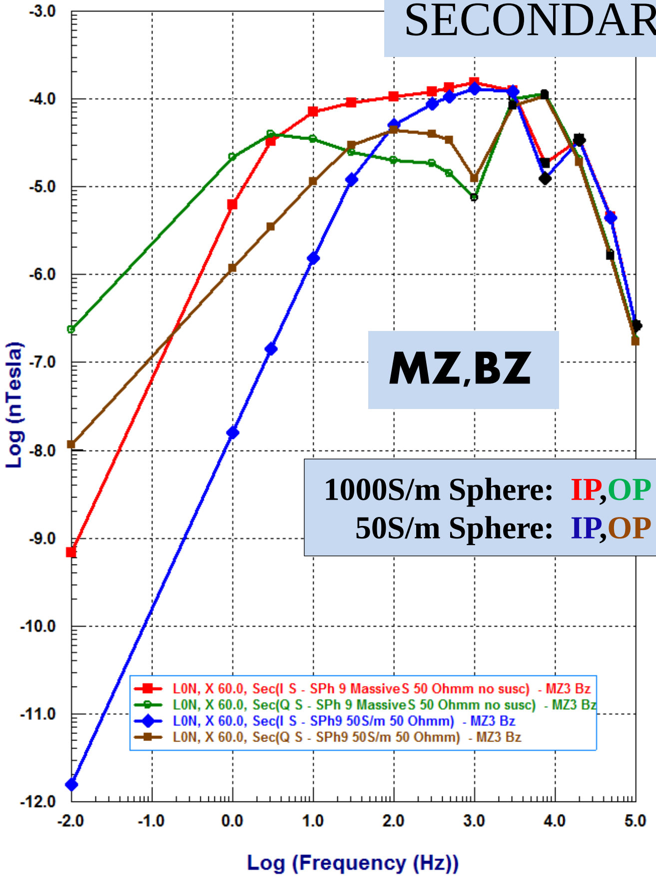


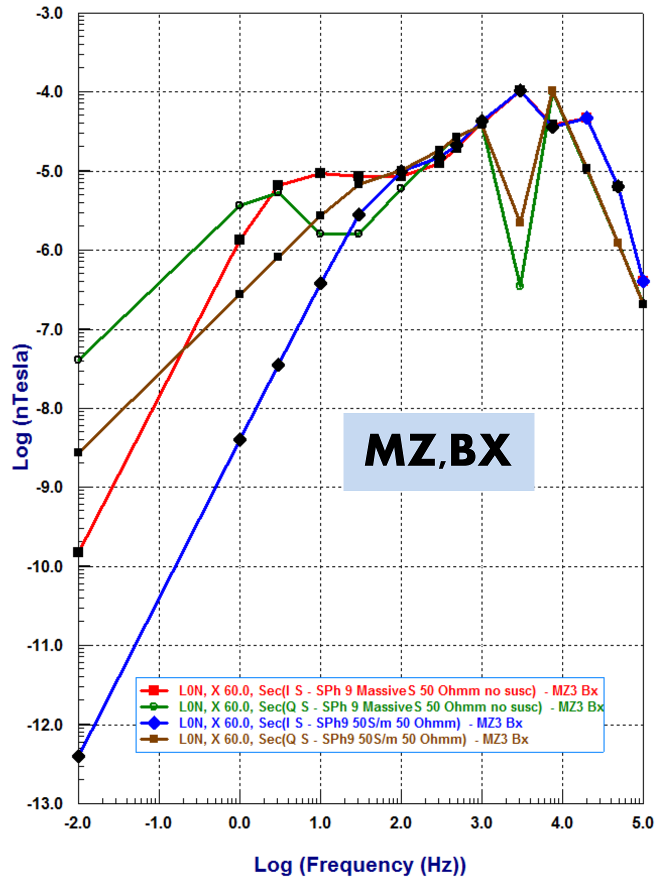
SPHERE: R=20m, (σ=1000,50),χ=[0,10],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm
3 Dipole Transmitters: (-60,0,0)

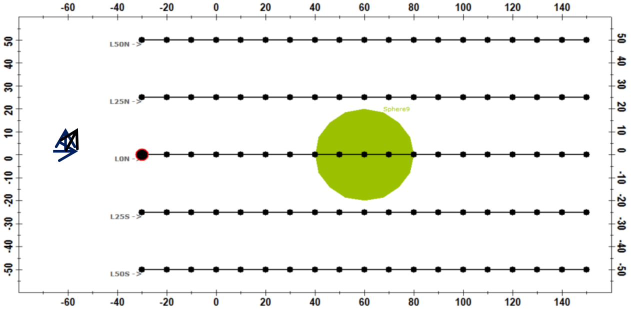
In this case, little to differentiate the 2 spheres when above 100Hz. Major differences are at frequencies far below most EM systems.

SECONDARY FIELDS





Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}



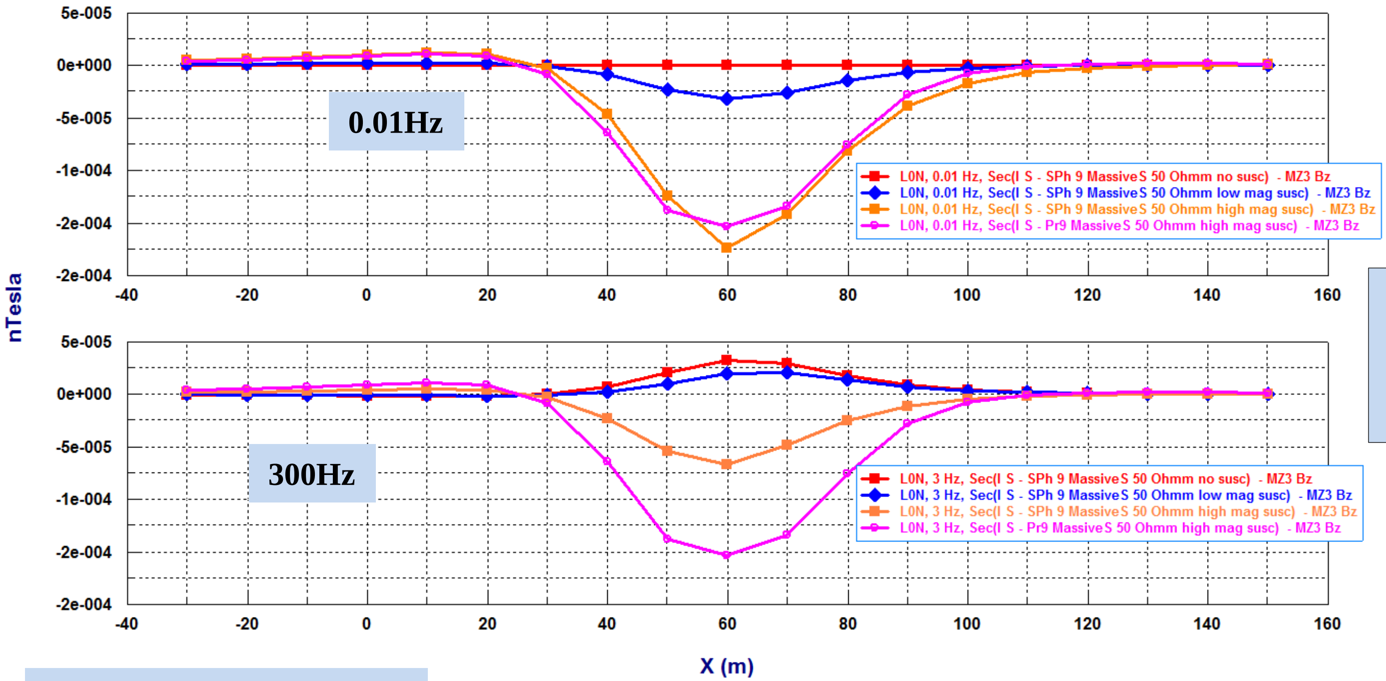
SPHERE: R=20m, (σ=1000),χ=[0,3,10],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm,3 Dipole Transmitters: (-60,0,0)

In this example, the TX's and sphere parameters are as in the previous examples The sphere is 1000S/m and varies in susceptibility (0,1,10). Now all effects are in place and to understand we illustrate at a few frequencies stating low and increasing in frequency.

Inphase MZ,BZ

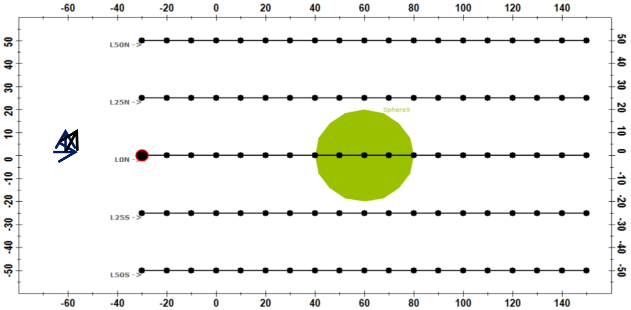
At very low frequencies (top), there is virtually no inductive response to even this massive conductor. Adding a little susceptibility causes a small magnetostatic response which increases with enhanced susceptibility. The LN response with the same conductivity and χ=10 matches the SPHERE response and this allows no inductive response. As the frequency increases, the inductive response increases and is positive and offsets the magnetostatic response which does not vary with frequency. However, the strong magnetization still significantly modifies the inductive response at 300Hz.



Sphere, χ=0: IP
Sphere, χ=3: IP
Sphere, χ=10: IP
LN , χ=10: IP

SECONDARY FIELDS

Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}



SPHERE: R=20m, (σ=1000),χ=[0,3,10],ε_r=1), Top=(60,0,-15)

Background Resistivity: 50Ωm,3 Dipole Transmitters: (-60,0,0)

In this example, the TX's and sphere parameters are as in the previous examples The sphere is 1000S/m and varies in susceptibility (0,1,10). Now all effects are in place and to understand we illustrate at a few frequencies stating low and increasing in frequency.

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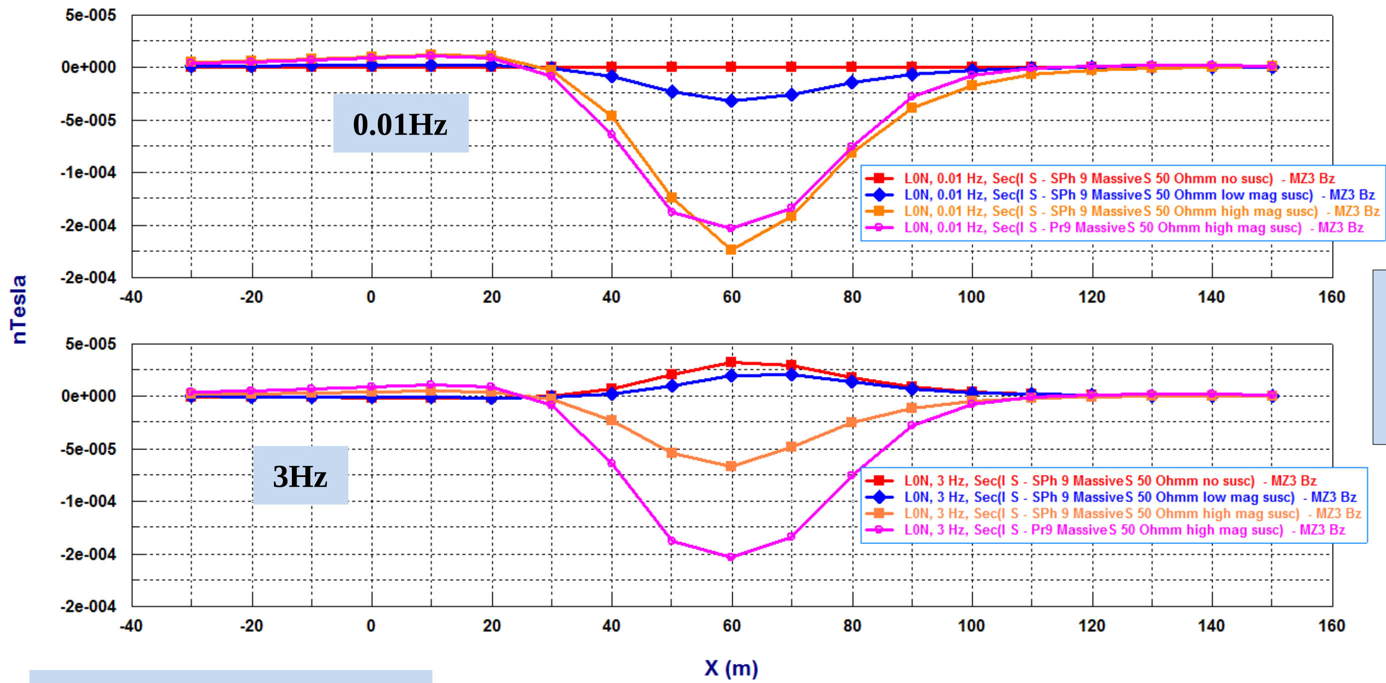


Sphere, χ=0: IP
Sphere, χ=3: IP
Sphere, χ=10: IP
LN , χ=10: IP

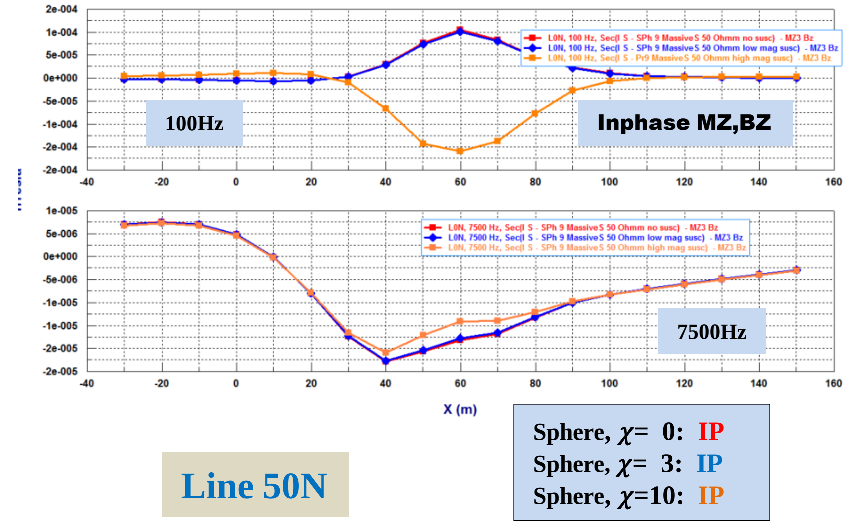
SECONDARY FIELDS

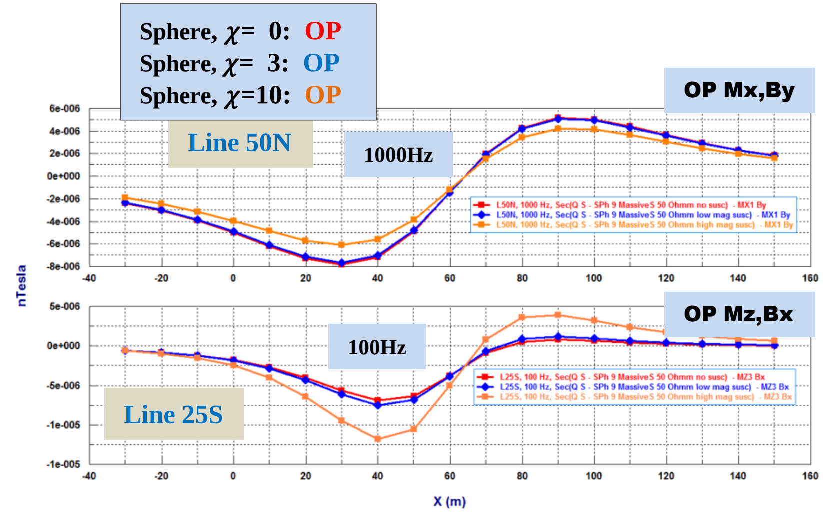
Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}

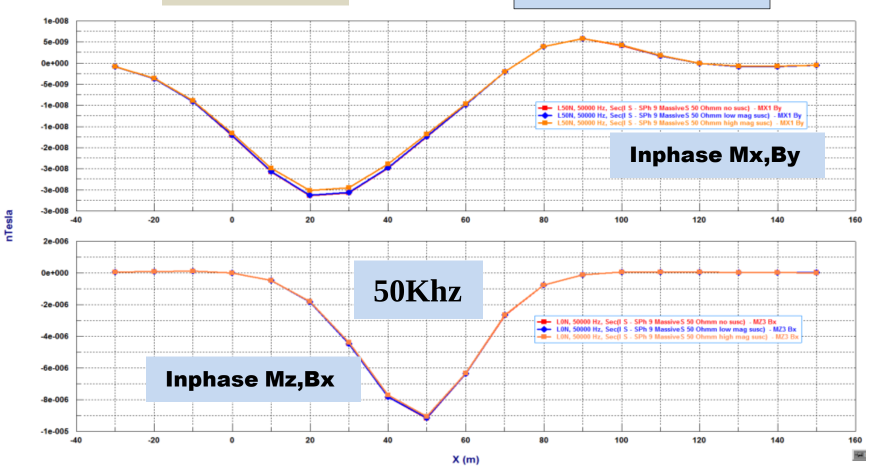
SECONDARY FIELDS

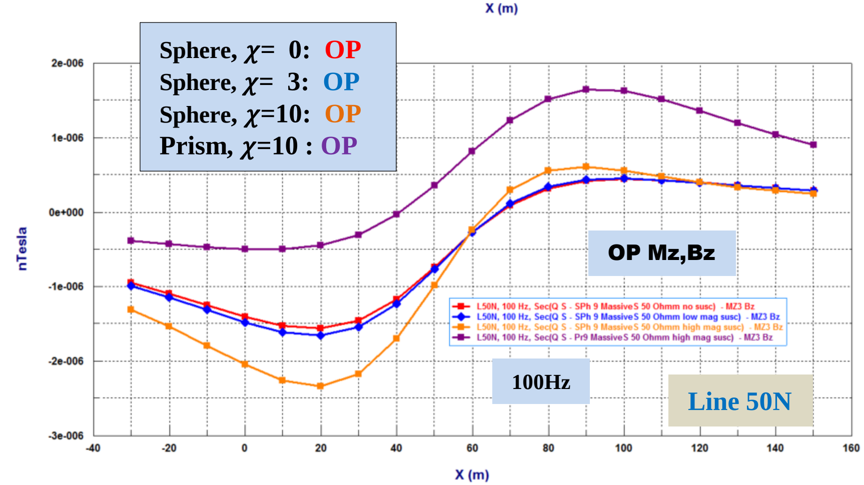
SPHERE: R=20m, (σ=1000),χ=[0,3,10],ε_r=1), Top=(60,0,-15)

At the top left, we see as the frequency increases (enhanced induction), the effects of magnetism on the IP becomes less. However, even at very high frequencies on a line further from the center line, the magnetism still has some effect. On the right, are various examples of the effects of magnetism on the quadrature response. On the bottom right, we see the separate effects of current channeling , induction and enhanced with magnetism present.

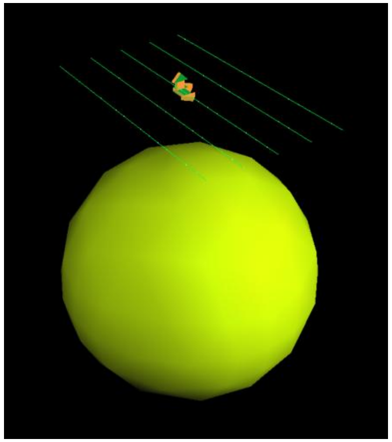


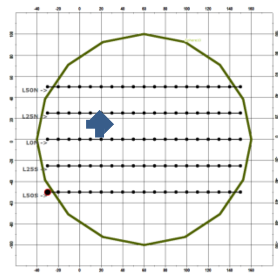






Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}





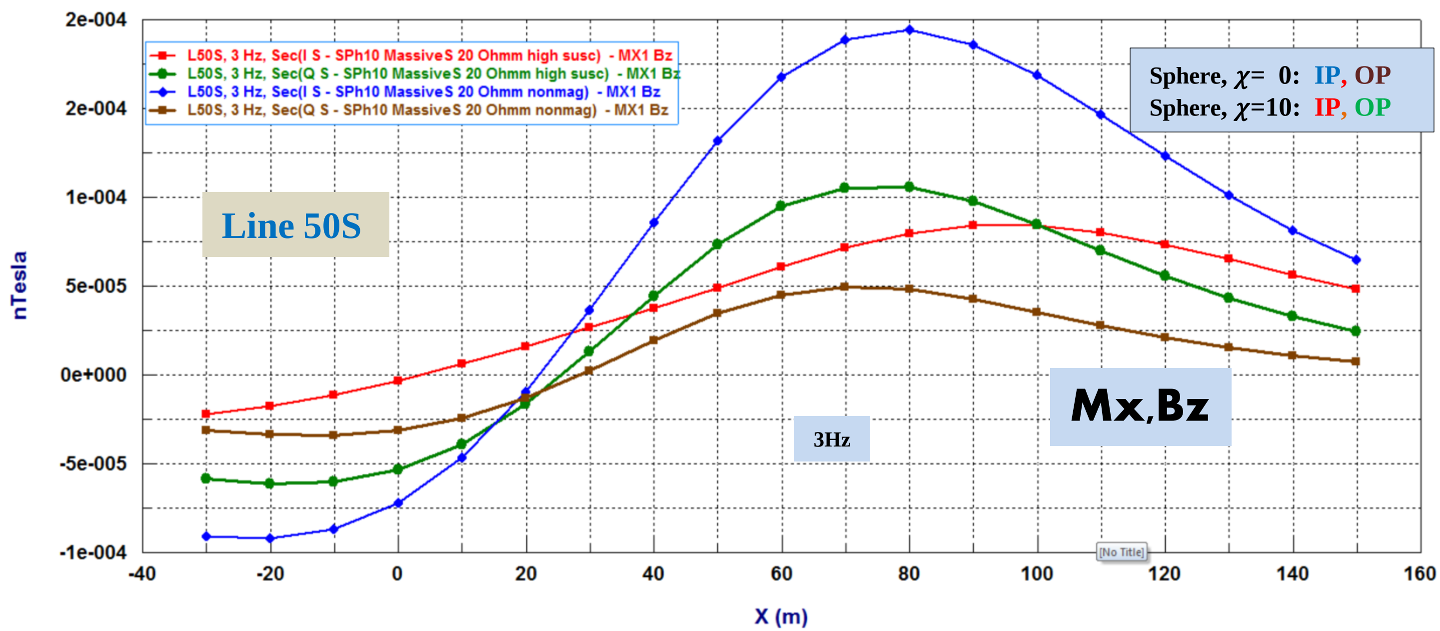
SPHERE: R=100m, (σ=1000),χ=[0,10],ε_r=1), Top=(60,0,-50)

Background Resistivity: 20Ωm
3 Dipole Transmitters: (30,10,0)

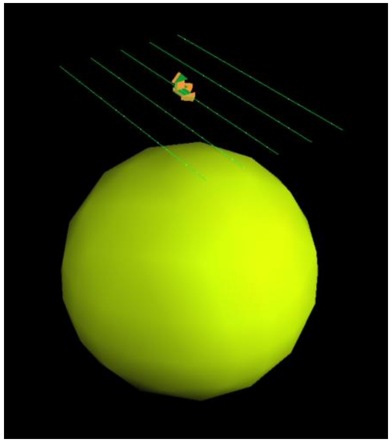
In this example, the top of the sphere is in the same location but the sphere radius is increased to 100m and the background is more conductive But, the major point is that the sources are not changed to be over the sphere which will change the nature of the inductive effects substantially and also to galvanic (current channeling responses. We look here at 2 cases, where the massively conductive sphere is magnetic and not magnetic.

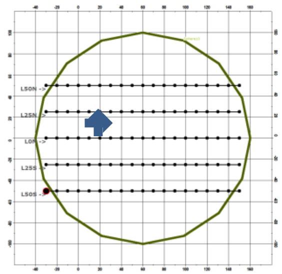
At this frequency and with this coupling, there is a strong affect from the magnetism in both the Inphase and Quadrature responses.

SECONDARY FIELDS



Numerical Examples by Model Simulation : Case 1 Far Transmitters: {strong conductor, magnetic/non-magnetic}



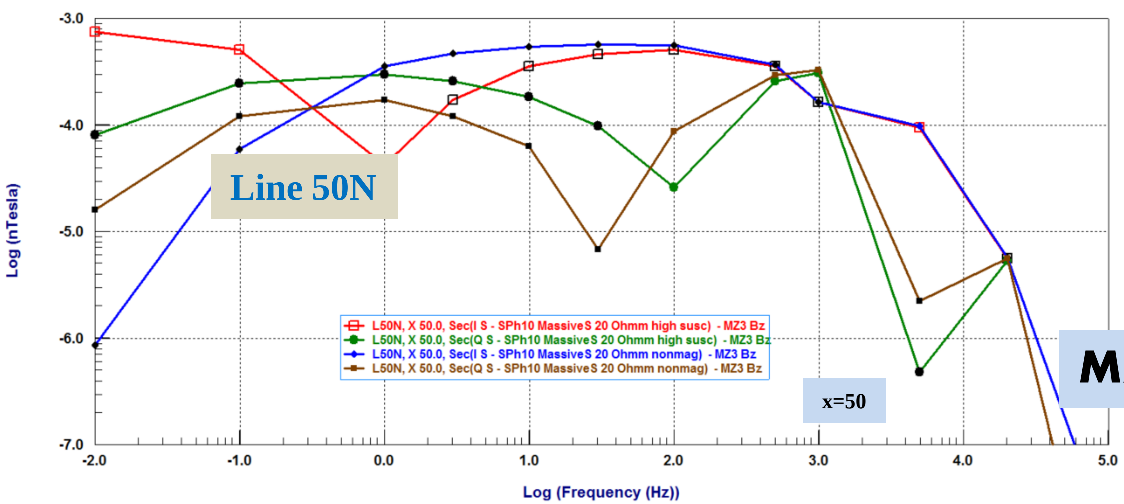


SPHERE: R=100m, (σ=1000),χ=[0,10],ε_r=1), Top=(60,0,-50)

Background Resistivity: 20Ωm
3 Dipole Transmitters: (30,10,0)

In this spectral response is indicated at this location and for this source and receiver component, the quadrature is influenced throughout the frequency range while the inphase for the magnetic and the non-magnetic come together around 500Hz. Note: the 100Khz solution did not converge for the magnetic target.

SECONDARY FIELDS



Sphere, χ= 0: IP, OP
Sphere, χ=10: IP, OP