Vertical Spatial Sensitivity and Exploration Depth of Low-Induction-Number Electromagnetic-Induction Instruments

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ABSTRACT

Vertical spatial sensitivity and effective depth of exploration (d_e) of low-induction-number (LIN) frequency-domain electromagnetic-induction instruments over a layered soil were evaluated using a complete numerical solution to Maxwell's equations. Previous studies using approximate mathematical solutions predicted a vertical spatial sensitivity for instruments operating under LIN conditions that, for a given transmitter-receiver coil-separation (s), coil orientation and transmitter frequency (ω), should depend solely on depth below the land surface. When not operating under LIN conditions, vertical spatial sensitivity and d_e also depend on apparent soil electrical conductivity (σ_a) and therefore the induction number (β) which is directly proportional to σ_a. In this new

evaluation, we determine the range of σ_a and β values for which the LIN conditions hold and how d_e changes when they do not. Two-layer soil models were simulated with both horizontal (HCP) and vertical (VCP) coplanar coil orientations, ω = 9,800 Hz and s = 3.66 m. Soil layers were given electrical conductivity values ranging from 0.1 to 200 mS m^{-1}. As expected, d_e decreased as σ_a increased. Only the least electrically conductive soil produced a d_e similar to that expected when operating under LIN conditions. For the VCP orientation, this was 1.6 s, decreasing to 0.8 s in the most electrically conductive soil. For the HCP orientation, d_e decreased from 0.76 s to 0.51 s. Differences between this and previous studies are attributed to inadequate representation by the approximate solution of: (1) skin-depth effect (attenuation and phase rotation of the fields traveling through the medium) and (2) scattering at interfaces between layers. It is thus important to consider the dependence of d_e on σ_a when using LIN FEM instruments to identify depth to targets such as water tables, interfaces between soil or lithologic layers and variations in salt or moisture content.

Abbreviations: σ, electrical conductivity; σ_a, apparent soil electrical conductivity; CS, cumulative sensitivity; d_e, effective depth of exploration; FEM, frequency-domain electromagnetic-induction; HCP, horizontal coplanar; LIN, low-induction-number; LS, local sensitivity; VCP, vertical coplanar

INTRODUCTION

Low-induction-number frequency-domain electromagnetic-induction (LIN FEM) instruments use the propagation of alternating electromagnetic fields through the soil to measure the apparent soil electrical conductivity (σ_a) [mS m^{-1}]. This measured property is

a complicated average of spatially distributed localized electrical conductivities in the subsurface. Apparent soil electrical conductivity is affected by several factors including water content, mineralogy, temperature, soil texture, porosity, permeability, and salinity. Instruments capable of operating as LIN FEM instruments include the EM38, EM31, and EM34 (Geonics Limited, Mississauga, ON, Canada), the DUALEM instruments series (DUALEM, Inc, Milton, ON) and the GEM instrument series (Geophex, Ltd., NC). The range of applications of LIN FEM instruments for environmental and hydrologic characterization and monitoring is large and ever increasing. Applications include aquifer extent and water content studies (António and Pacheco, 2002; Schneider and Kruse, 2003; Sheets and Hendrickx, 1995), and lithology, soil salinity, and soil texture mapping (Benjoudi et al., 2002; Lesch et al., 1998; Paine 2003; Stroh et al., 2001; Sudduth et al., 2005; Triantafilis et al., 2005; Yoder et al., 2001). LIN FEM instruments also have been used to delineate landfills (Lanz et al., 1998; Nyquist and Blair, 1991), contaminant plumes (Matias et al., 1994), and areas of active recharge (Salama et al., 1994).

Quantitative applications of LIN FEM instruments to hydrologic investigations depend on the ability to transform measured electrical conductivities into vertical and horizontal variations of hydrologically relevant properties. The accuracy of these transformations relies, primarily on (1) calibrating electrical conductivity to a property of interest (e.g. volumetric water content) either through qualitative or numerical inversion, and (2) mapping interpreted hydrologic variables based on measurement locations. A comprehensive review of modeling and inversion as applied to electric and electromagnetic methods was recently published by Pellerin and Wannamaker (2005). In the present investigation, we exclusively examine the latter, less commonly considered

issue. Specifically, we use forward numerical models of layered soils to study the effect of variations in σ_a on the vertical spatial sensitivity of LIN FEM instruments. This information is useful when attempting to assign depths to properties such as soil water content (e.g. depth to the water table), soil texture, salinity, and hydraulic conductivity.

It is commonly assumed that, under most conditions, the sample depth (or sample volume in three-dimensions) of LIN FEM instruments is independent of the subsurface electrical conductivity. McNeill (1980) defined "effective depths of exploration" (d_e) based on the vertical spatial sensitivity of LIN FEM instruments in homogeneous and horizontally layered soils, which he computed using an asymptotic approximation of Maxwell's equations. The asymptotic approximation is based on the assumption that the induction number (β) is very small. (As such, this solution is sometimes referred to as the "LIN approximation", and it formed the basis for development of many LIN FEM instruments.) The induction number is the ratio of the intercoil separation (s) (Fig. 1) to the skin depth (δ) (Kaufman and Keller, 1983; Spies, 1989):

$$\beta = \frac{s}{\delta} = \frac{s}{\sqrt{\frac{2}{\sigma_a \omega \mu}}} \qquad [1]$$

Where ω is angular frequency [s^{-1}], and μ is magnetic permeability [Tm A^{-1}] and μ normally is assumed to be constant and equivalent to its free-space value (μ_0). Skin depth is the depth at which the transmitted magnetic field strength has decayed to e^{-1} of its initial magnitude at a reference point. In the low frequency limit, δ varies inversely with the square root of σ_a. As σ_a increases, δ decreases and β increases, effectively decreasing instrument sensitivity with increasing depth, and decreasing d_e. Some authors used the LIN approximation in their investigations, but did not state the range of β required for a

valid approximation. McNeill (1980) asserted that the LIN approximation holds where β is much less than 1 and recommended that LIN FEM instruments be used in environments where $\sigma_a \leq 100$ mS m^{-1} (which for ω = 9,800 Hz and s = 3.66 m corresponds to $\beta \leq 0.23$). Wait (1962) defined this as β less than about 0.3 and Frischknecht (1987) defined the criterion as β less than 0.02. This disagreement makes it unclear under what values of β the LIN approximation can be applied.

In this study, we use a standard commercial forward numerical code of electromagnetic field propagation (EMIGMA, PetRos EiKon, Inc. 2004) to examine the d_e of FEM instruments. A numerical code does not require the LIN approximation, and is therefore more accurate in its representation of physical processes affecting electromagnetic wave propagation and d_e of FEM instruments. A numerical code also allows for identifying conditions under which the LIN approximation might not be applicable. Specifically, like McNeill (1980), we define the vertical spatial sensitivity of an FEM instrument in horizontally layered soil. Additionally, we vary σ_a to determine at what β values the LIN approximation holds and when it does not, to determine the effect on d_e. Based on our findings, we re-define the conditions under which the LIN approximation should be used.

THEORY

LIN FEM instruments operate use with a transmitter coil to generate an alternating magnetic field ($\partial \boldsymbol{H}/\partial t$) where $\boldsymbol{H}$ is the magnetic field strength [A m^{-1}], t is time [s], and bold H is a vector. This magnetic field propagates into the subsurface where it induces alternating currents as described by Faraday's law:

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \quad [2]$$

where $\boldsymbol{E}$ is electric field strength [V m^{-1}]. This equation states that a time varying magnetic field will produce an electric field whose curl is equal to the negative of the time derivative of the magnetic field. The electric field induces currents in the soil, which, in turn, produce their own magnetic fields whose polarity and magnitude oppose the change in the incident field. The incident magnetic field often is called the primary field and the field generated by the induced currents is called the secondary field. These two fields combine to induce currents in the receiver coil in accordance with Faraday's law. The transmitter and receiver coils, and associated magnetic dipoles, can be oriented relative to each other and to the soil surface. Orientations considered in this study were horizontal coplanar (both coils lie flat on the ground) and vertical coplanar (coils are upright and coplanar) (Fig. 1).

Maxwell's equations generally describe the macroscopic behavior of electromagnetic fields (Gomez-Treviño et al., 2002). Numerical as well as analytical and approximate solutions to these equations can be used to investigate the propagation of subsurface electromagnetic fields under specific conditions. Typically, analytical solutions can be derived only for simple scenarios like uniform half-spaces (Ward and Hohmann, 1988). Even with geometric simplifications, additional restrictive simplifying assumptions are required to solve the equations analytically. Approximate solutions, such as the solution used by McNeill (1980), use restrictive physical and mathematical assumptions to arrive at solutions that are approximately valid in some limited physical regime. Numerical codes developed in the 1980s (Anderson, 1984; Tabbagh, 1985; Wannamaker et al., 1984) allow for consideration of more complex, realistic soil models.

In the development of EMIGMA, the code used for this study, an attempt was made not to simplify the mathematics in any way. Although small numerical errors still result from the calculations, errors in EMIGMA are generally less than errors expected from instruments or geologic noise. In addition to allowing for more general geometries, numerical codes can more accurately represent processes such as skin depth affects and scattering. Scattering, which includes reflection, refraction, current channeling, and induction, describes the distortion of electromagnetic waves at spatial discontinuities in electrical and magnetic properties of soil. These various processes affect propagation of the incident electromagnetic field and consequently the propagation and magnitude of secondary electromagnetic fields.

McNeill's (1980) white paper discussed part of the theory and assumptions of the mathematical approximations that provide the basis for development, use, and interpretation of LIN FEM instruments. The derivation of equations discussed in McNeill's paper was developed in greater detail by Belluigi (1949), Wait (1955, 1962), Kaufman and Keller (1983), and Gomez-Treviño et al. (2002). The LIN approximation is derived from Maxwell's equations for a one-dimensional soil (homogeneous, layered, or arbitrary electrical conductivity (σ) variations with depth) in which transmitter frequency is low and the distance (s) between a transmitting magnetic dipole and a receiver (Fig. 1) is small compared to the skin depth (δ) [m] of magnetic fields in the soil (LIN conditions). For these conditions, vertical spatial sensitivity and d_e are independent of σ_a and the LIN approximation is valid. However, opinions on what constitutes LIN conditions can vary by investigator. For an EM31 instrument with a coil spacing of 3.66 m and an operating frequency of 9,800 Hz, the LIN assumptions might be valid for σ_a

<200 mS m^{-1} for Wait's (1962) criterion, ≤100 mS m^{-1} for McNeill's (1980), or < 0.8 mS m^{-1} for Frischknecht's (1987). Table 1 shows the corresponding values for resistivity and skin depth. A practical difference exists between 0.8 mS m^{-1} and the two higher values. Electrical conductivity of most surficial material in the continental United States likely lies between 0.8 and 100 mS m^{-1} (Keller and Frischknecht, 1966). If either McNeill's or Wait's criterion is correct, then the LIN approximation and McNeill's associated guides to data interpretation should be able to be used without significant problems. If Frischknecht's is the correct criterion, however, soil and rock electrical conductivities at most field sites likely are too high for the LIN approximation to be valid. As a result, inferences of true σ_a, and thicknesses and numbers of soil layers based on these assumptions could be incorrect.

When site characteristics and instrument parameters combine to satisfy the LIN conditions, the quadrature component of the secondary magnetic field strength, H_s, is linearly dependent on σ_a (McNeill, 1980). The quadrature component of the secondary magnetic field is that part of the secondary magnetic field that is 90° out of phase with the primary field (Telford et al., 1990).:

$$\sigma_a = \frac{4}{\omega \mu_o s^2} \frac{(H_S)_{\substack{Quadrature \\ Component}}}{H_P} \qquad [3]$$

Where H_p is the primary magnetic field strength. For a given measurement of σ_a, coil orientation, s and ω usually are fixed; however if s is increased or ω decreased, d_e increases. The effective depth of exploration will also increase by changing the magnetic dipole orientation from horizontal (HCP) to vertical (VCP), because VCP sensitivity distribution peaks at greater depths than the HCP distribution (Fig. 2).

McNeill (1980) provided a simple form of spatial sensitivity analysis using his "cumulative response", which we change to "cumulative sensitivity", (a measure of an instrument's distribution of sensitivity in the subsurface). Cumulative sensitivity (*CS*) can be used to determine sensitivity of LIN FEM instruments to all material above or below a given depth. Depths are normalized to facilitate comparisons of instruments with different intercoil separations. The normalized depth, z, is the actual depth divided by the intercoil separation, s, and is referred to in terms of s. For instance, a 7.32 m depth for an instrument with a 3.66-m intercoil separation is referred to as 2 s. Equations of cumulative sensitivity for the horizontal (CS_{HCP}) and vertical (CS_{VCP}) coil orientations as a function of normalized depth (McNeill, 1980) are:

$$CS_{HCP}(z) = \int_{z}^{\infty} LS_{HCP}(z)dz = \frac{1}{(4z^2+1)^{1/2}} \quad [4]$$

$$CS_{VCP}(z) = \int_{z}^{\infty} LS_{VCP}(z)dz = (4z^2+1)^{1/2} - 2z \quad [5]$$

where local sensitivity (*LS*) is the relative contribution of material at a given depth to the measured value of σ_a. A *CS* value represents the fraction of the secondary field at the receiver that originates between a depth z and infinite depth. When z is small and *CS* is near 1, most of the measured response comes from soil at a depth greater than z. The part of the response due to material in the interval between the surface (z = 0) and z is 1-*CS*. By plotting the functions in Eqs. [4] and [5], and locating the *CS* value associated with McNeill's exploration depths, it appears that McNeill used a *CS* value of 0.7 (or 70 percent) to define d_e (McNeill, 1980, p. 5-6). For example, when *CS* = 0.3 for HCP orientation, about 30 percent of the measured response is attributable to material at depths greater than 0.76 s (Fig. 2). Therefore, the interval between 0 s and 0.76 s contributes 70

percent of the HCP orientation response. Material located between 0 and 1.6 *s* contributes 70 percent of the VCP orientation response (Fig. 2). These depths, 0.76 *s* and 1.6 *s*, represent d_e for these orientations under LIN conditions.

In a two-layer soil, for which *CS* is calculated from infinite depth to the contact between the layers, the top layer's contribution to the measured response is 1-*CS*. In such soil, *CS* near 1 represents a nearly homogeneous soil where σ_a is equal to σ of the lower layer. Cumulative sensitivity near 0 represents a nearly homogeneous soil where σ_a is equal to σ of the upper layer. In a two-layer soil, σ_a can be calculated by:

$$\sigma_a = \sigma_{Layer1}(1 - CS) + \sigma_{Layer2} CS \qquad [6]$$

This calculation is equivalent to equation 5 from McNeill (1980). The cumulative sensitivity can be determined based on σ_a of a two-layered soil if σ values of each layer (σ_{Layer1}, σ_{Layer2}) are known. Specifically, rearranging Eq. [6] to solve for *CS* gives:

$$CS = \frac{\sigma_a - \sigma_{Layer1}}{\sigma_{Layer2} - \sigma_{Layer1}} \qquad [7]$$

METHODS

In this study, the software EMIGMA (PetRos EiKon Inc., 2004) was used to conduct forward numerical simulations of a Geonics EM31 instrument response over a nearly homogeneous or horizontally layered soil. The software was developed to calculate three-dimensional, electromagnetic-field propagation in complex environments for a great variety of instrument systems. The code, prior to commercial release, was calibrated against a number of academic and internal industry codes, as well as codes published in several articles and doctoral theses. An intercoil separation of 3.66 m and a

transmitting frequency of 9,800 Hz were used for all simulations. These are the instrument parameters of used in Geonics EM31 instrument. The software represents coils as alternating magnetic dipoles which for these simulations were placed 0.05 m above ground. Magnetic permeability was constant for all simulations and set equal to its free-space value (μ_0). The ratio of the secondary to the primary magnetic field magnitude, in percent, was used as output format instead of σ_a, because this is the more common output format of many instruments. Moreover, σ_a is inherently less accurate than the magnetic field ratio because it can only be calculated through the use of numerical inversion routines or approximate models.

All results are presented as *CS* for direct comparison with results of LIN approximation. We simulated two-layer soils differing only in σ of the layers. Individual-layer electrical conductivities ranged from 0.1 to 200 mS m^{-1}. In each two-layered soil, upper and lower layer σ varied from nearly equal (e.g. upper layer 99 mS m^{-1}, lower layer 100 mS m^{-1}), to very different values (e.g. upper layer 100 mS m^{-1}, lower layer 0.1 mS m^{-1}). For each σ-value used in the simulations, the response (ratio of magnetic field magnitudes) was determined for a homogeneous half-space for both horizontal- and vertical-coplanar coil orientations. A series of simulations was then carried out by varying upper layer thickness. Equation 7 was used to determine the numerical *CS* value from the simulation results. One value in the *CS* distribution was calculated from each simulation. Each *CS* value was plotted as a function of depth to the interface between layers to generate a curve for comparison with curves in figure 2. Comparing LIN-approximation curves facilitated determining the effects of changing individual-layer electrical conductivities on the vertical spatial sensitivity distribution.

Cumulative sensitivity was calculated for two soil-model types: Type 1 and Type 2 (Fig. 3). For a given series of simulations, a constant-σ upper layer was used for Type 1 soil models and constant-σ lower layer was used for Type 2 soil models. Each model type was evaluated at two ranges of σ_a, one electrically "resistive" and one electrically "conductive", in which the soil model was composed of two layers and thickness of the top layer ranged from 0.001 to 18 m (or 0 to 4.9 *s*). For all soil models, the bottom layer was essentially infinitely thick (1×10^8 m). For Type 1 resistive soil models, σ of the upper layer was held constant at 0.1 mS m^{-1} and σ of the lower layer was given one of three values: 0.2, 10, or 100 mS m^{-1}. For Type 2 resistive soil models, σ of the upper layer was given values of 0.2, 10, or 100 mS m^{-1}, and σ of the lower layer was held constant at 0.1 mS m^{-1}. These resistive soil models were used to match the LIN approximate solution and to study vertical sensitivity distribution changes in more electrically resistive environments. To mimic conditions common in near-surface investigations, a series of electrically conductive simulations were carried out at higher σ_a values using the same Type 1 and 2 soil-model scenarios. Such conditions might be found at waste sites contaminated with inorganic compounds, at landfills or in saturated clays. The constant-σ layer was 100 mS m^{-1} and five soils were simulated in which the variable layer σ ranged from 0.1 to 200 mS m^{-1}.

RESULTS AND DISCUSSION

Cumulative sensitivity was plotted as a function of depth to the interface between upper and lower layers to compare the vertical sensitivity distribution of the LIN approximation over a layered soil with those derived from numerical simulations (Figs.

4-6). Except for two soils with a 200 mS m^{-1} layer, σ in all simulated soils was less than or equal to 100 mS m^{-1} (one of McNeill's (1980) criteria for the LIN conditions). A soil model in which upper layer σ was 0.1 mS m^{-1} and lower layer σ was 0.2 mS m^{-1} is referred to as the [0.1/0.2] soil model. Induction numbers calculated from the results of simulations ranged from a minimum of about 0.01 for the [0.1/0.2] and [0.2/0.1] soil models to between 0.23 and 0.32 for the [100/200] and [200/100] soil models (Table 1). One conclusion of the LIN approximation is that vertical sensitivity, and therefore d_e, does not depend on σ_a. However, results of simulations indicate that instrument spatial sensitivity varies significantly with changes in σ_a. This was true for Type 1 and 2 models for both homogeneous, (e.g., the [0.1/0.2] and [100/99] soil models), as well as heterogeneous soil models. The general shape of all cumulative sensitivity distributions was similar to that predicted by the LIN approximation. In the more electrically resistive soils simulated, [0.1/0.2] and [0.2/0.1], *CS* values were close to those predicted by the LIN approximation (Fig. 4). In soils with higher σ_a, however, the simulated *CS* values were at shallower depths than comparable LIN approximation values. For example, in Fig. 4A, the HCP LIN approximation predicts a *CS* value of 0.2 will occur at a depth of about 2.5 *s*. In contrast, the numerical approach predicts that as σ increases, the *CS* depth decreases from just under 2.5 *s* for the [0.2/0.1] and [0.1/0.2] soil models to 2.0, 1.8, and 1.5 *s* for the [0.1/10], [10/0.1], and [0.1/100] soil models, respectively. The minimum *CS* depth was about 1.2 *s* for the [100/0.1] soil model. Although nearly all simulated σ_a values are within the LIN range indicated by McNeill (1980) and Frischknecht (1987), only simulations of the [0.1/0.2] and [0.2/0.1] soil models ($\beta \approx 0.01$) came close to the vertical sensitivity distribution predicted by the LIN approximation (Fig. 4). This

indicates that only for electrically resistive soils do the LIN approximation and its predictions of the effective depth of exploration hold. Thus for many soils, depth to targets such as the water table or a particular soil horizon may be misjudged even at moderate values of σ_a. For electrically conductive soils, for instance those that are clay-rich, salty or wet, such targets may be missed entirely.

There were a number of similarities and differences between the results of Type 1 and 2 soil-model simulations. In both model types, lower *CS* values occurred in soil with the higher upper-layer σ. For instance, in the [0.1/10] and [10/0.1] soil models, lower *CS* values were in the [10/0.1] soil model. This indicates that a LIN instrument would be less sensitive at depth to a soil with a [10/0.1] layering than to one with a [0.1/10] layering. Thus, it may be difficult to distinguish temporal or spatial variability of σ_a, for example while tracking changes in salt or moisture content, below an electrically conductive surface layer. For a given σ-contrast, soil models with varied lower-layer σ (Type 1) were more similar to one another than soil models with varied upper-layer σ (Type 2). Deeper layers seemed to have less effect on VCP results than on HCP results, probably because the VCP orientation was more sensitive to shallow depths than HCP orientation. For both orientations, Type 1 curves are more similar to one another than Type 2 curves, possibly because the source of variability, the top layer, is closer to the instrument. The last similarity to note is that, at any given depth, a non-linear decrease in *CS* values occurred as the electrical conductivity of either layer increased. All these trends also occurred in higher conductivity soil models (Figs. 5 and 6).

McNeill's (1980) criterion for identifying d_e was $CS = 0.3$. Using this criterion for each numerical model, we determined d_e by locating the depth on each curve that

corresponded to a *CS* value of 0.3 (Figs. 4-6). In contrast to previous findings, d_e decreased with increasing σ_a (Fig. 7). This deviation from the LIN approximation was found in all soils simulated, including homogeneous and heterogeneous soils. The effect of increasing electrical conductivity on d_e was different for each coil orientation. For the VCP orientation, d_e for all numerical models tested ranged from 1.6 *s*, the same depth as the LIN approximation, to about 0.8 *s* for the most electrically conductive soil. Using the VCP LIN d_e could result in an overestimate of depth to soil-layer interfaces by as much as 100 percent of the actual d_e. For the HCP orientation, d_e ranged from 0.76 *s* to 0.51 *s*. Using the VCP LIN d_e could result in an overestimate of as much as 50 percent. Deviations of 10 percent or more from the LIN predicted depths occurred at σ_a values of 3 mS m^{-1} or greater. Given the range of near-surface σ_a in the continental USA (Keller and Frischknecht, 1966), reliance on the LIN assumptions at many field sites could lead to significant overestimates of d_e and poor estimates of layer thickness and composition, water content or depth to interfaces such as the water table or changes in soil texture.

Considering the range of β, differences between the LIN and numerical approaches likely are caused by skin affect and scattering phenomena, which are not adequately represented by the LIN approximation. Scattering can affect both primary and secondary magnetic fields. Both processes affect the cumulative secondary field sensed by the instruments at the surface. As a result, vertical variations in electrical conductivity will affect the results of field surveys by changing vertical spatial sensitivity. Temporal variations in water content, soil temperature, and salinity cause temporal variations in σ_a, which may change d_e. At any field site where physical, chemical, or hydraulic properties are spatially non-uniform, d_e is also likely to be non-uniform. Although the numerical

data plotted in figure 7 will allow for a better understanding of the behavior d_e with change in σ_a, accurate determination of d_e may be difficult without the aid of numerical simulations.

CONCLUSIONS

Vertical spatial sensitivity and the effective depth of exploration of low-induction-number frequency-domain electromagnetic (LIN FEM) instruments in two-layer soils were studied using an industry-standard numerical model. The model simulation results were compared to interpretations based on the LIN asymptotic approximation developed previously. Clear differences were indicated between the predictions of the approximate and numerical approaches. In the numerical simulations, the effective depth of exploration decreased by up to 50 percent as the electrical conductivity of the soil increased when compared with the predictions of the LIN approximation. In the LIN approximation, vertical spatial sensitivity and effective depth of exploration are independent of the apparent soil electrical conductivity. In contrast, the numerical models indicate that magnitude and distribution of soil properties affecting electrical conductivity in the subsurface can significantly alter LIN instruments' vertical spatial sensitivity and consequently their effective depth of exploration. In electrically resistive soil models, where induction numbers were small (induction number ≤ 0.01 for a 0.1 mS m^{-1} upper layer over 0.2 mS m^{-1} lower layer soil model), the vertical sensitivity distribution was similar to that predicted using the LIN asymptotic approximation. Under more electrically conductive conditions, even those that meet the low-induction-number criteria of previous authors, a lower cumulative sensitivity than that predicted by the LIN

approximation was observed at a given depth. The results indicate that in hydrologic and environmental investigations in any but the most electrically resistive environments, LIN FEM instrument sensitivity will likely be focused closer to the surface than predicted by the LIN approximation. This has important implications for studies of soil or geologic layering, water table detection, studies of soil salinity, contaminant transport and landfill delineation. In short, all studies that use LIN FEM instruments to study the vertical distribution of properties in the subsurface need to incorporate this information into project planning and data analysis. More heterogeneous subsurface conditions will benefit from numerical simulation of electromagnetic wave propagation to aid in the attribution of hydrologic properties, inferred from LIN FEM measurements, to appropriate subsurface locations. However, when access to software, time or resources does not allow for numerical modeling, our plot of effective depth of exploration versus apparent soil electrical conductivity may be used to estimate the effective depth of exploration based on reasonable estimates of the subsurface electrical conductivity.

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Vertical Spatial Sensitivity and Exploration Depth of Low-Induction-Number Electromagnetic-Induction Instruments

James B. Callegary[a,*], Ty P.A. Ferré[b] and R.W. Groom[c]

FIGURE and TABLE CAPTIONS

Table 1. Induction number calculated for electrical conductivities with low-induction-number condition criteria proposed by several researchers.

Figure 1. LIN FEM instruments, two coils with an intercoil separation (*s*). One coil transmits (Tx) the primary magnetic field and one coil receives (Rx) a combination of the primary and secondary fields. Coils may be oriented either horizontally or vertically with respect to ground surface as indicated by directional arrows.

Figure 2. Cumulative sensitivity and relative depth to interface between layers, McNeill (1980). Approximations are for vertical (VCP) and horizontal (HCP) coplanar coil orientations. McNeill's "effective depth of exploration" is indicated by a cumulative sensitivity value of 0.3. Cumulative sensitivity of low-induction-number (LIN) instruments is calculated from infinity to a given relative depth. Relative depth is depth divided by intercoil separation.

Figure 3. Each set of cumulative-sensitivity simulations was based on a two-layer soil. Electrical conductivity of the upper layer in soil model Type 1 and the lower layer in soil model Type 2 were fixed at 0.1 mS m^{-1} for the electrically resistive case or 100 mS m^{-1} for the electrically conductive case. The variable electrical-conductivity layer was given values ranging from 0.1 to

200 mS m^{-1}. For a given two-layer soil model in each set of simulations, the thickness of the upper layer increased from 0.001m to 18 m. Tx indicates transmitter coil; Rx indicates receiver coil; and *s* is intercoil separation.

Figure 4. Cumulative sensitivity for soil model Types 1 and 2 calculated from forward numerical simulation results compared with low-induction-number (LIN) approximation. (A) horizontal (HCP) and (B) vertical (VCP) coplanar coil orientation. Relative depth is depth divided by intercoil separation. The first number in brackets is the upper layer electrical conductivity, in mS m-1; the second number is lower layer electrical conductivity. In (B), to show underlying LIN VCP approximation curves, the lines for the VCP [0.1/0.2] and [0.2/0.1] soil models were removed.

Figure 5. Soil model Type 1 cumulative sensitivity using the higher electrical conductivities sometimes detected in environmental investigations. Comparison of results of forward numerical simulation results with the low-induction-number (LIN) approximation. (A) is horizontal (HCP) and (B) is vertical (VCP) coplanar coil orientation. Relative depth is depth divided by intercoil separation. The first number in brackets is the upper layer electrical conductivity, in mS m^{-1}, the second number is that of the lower layer.

Figure 6. Soil model Type 2 cumulative sensitivity using the higher electrical conductivities sometimes detected in environmental investigations. Comparison of results of forward numerical model simulation results with the low-induction-number (LIN) approximation. (A) is horizontal (HCP) and (B) is vertical (VCP) coplanar coil orientation. Relative depth is the depth divided by the intercoil separation. The first number in brackets is the upper layer electrical conductivity, in mS m-1, the second number is the lower layer electrical conductivity.

Figure 7. Relation between apparent electrical conductivity and effective depth of exploration: comparison of simulated and LIN approximation results. VCP, vertical coplanar coil orientation; HCP, horizontal coplanar coil orientation. Effective depth of exploration is the depth divided by the intercoil separation.

1

Apparent soil electrical conductivity, σ_a (mS m^{-1})	Resistivity, σ^{-1} (ohm-m)	Skin depth, δ (m)	Induction number, β (--)	LIN condition criterion ($\beta \leq$ x) proposed by:
10,000	0.1	2	2.3	n/a
1,000	1	5	.72	n/a
500	2	7	.51	n/a
200	5	11	.32	Wait (1962)
100	10	16	.23	McNeill (1980)
50	20	23	.16	n/a
20	50	36	.10	n/a
10	100	51	.072	n/a
5	200	72	.051	n/a
2	500	114	.032	n/a
0.8	1,300	183	.020	Frischknecht (1987)
0.2	5,000	359	.010	n/a
0.1	10,000	508	.007	n/a
0.01	100,000	1,608	.002	n/a
Frequency (Hz)			9,800	
Coil Spacing (m)			3.66	
Magnetic permeability of free space (H m^{-1})			$4\pi \times 10^{-7}$	

2

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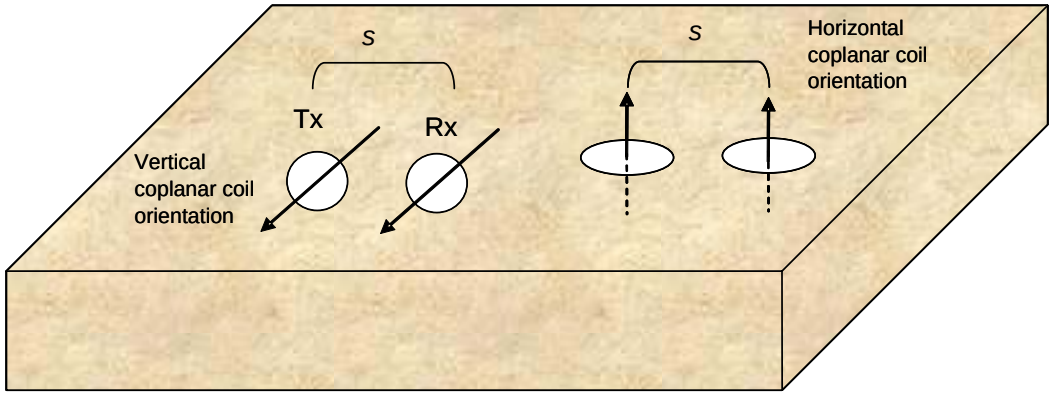
5



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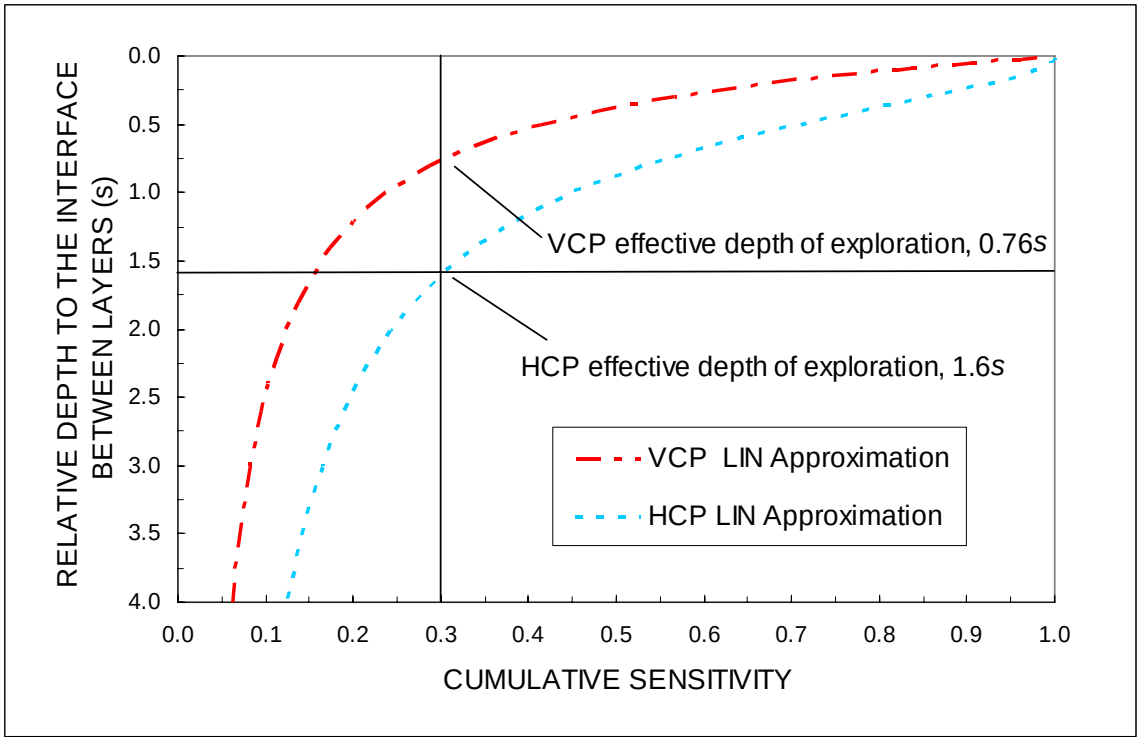

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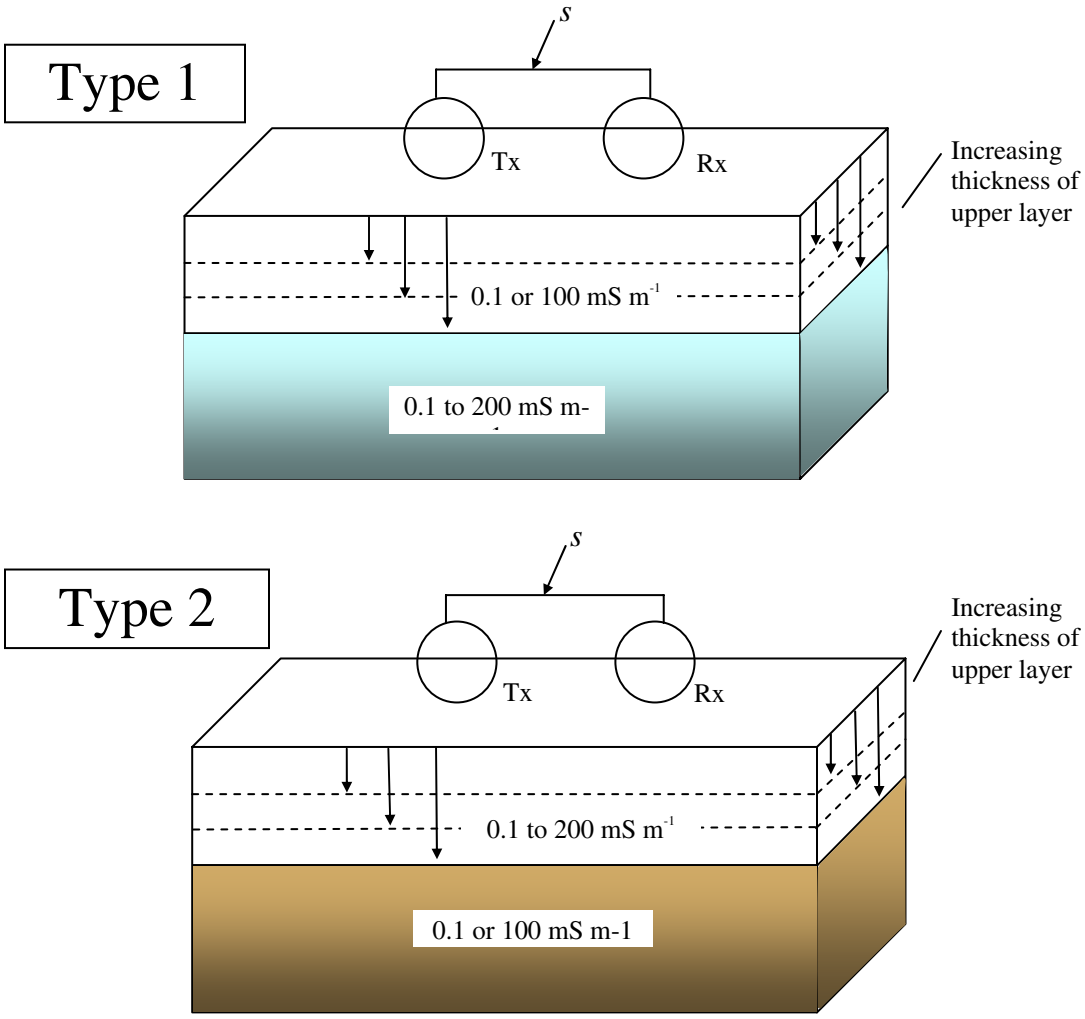

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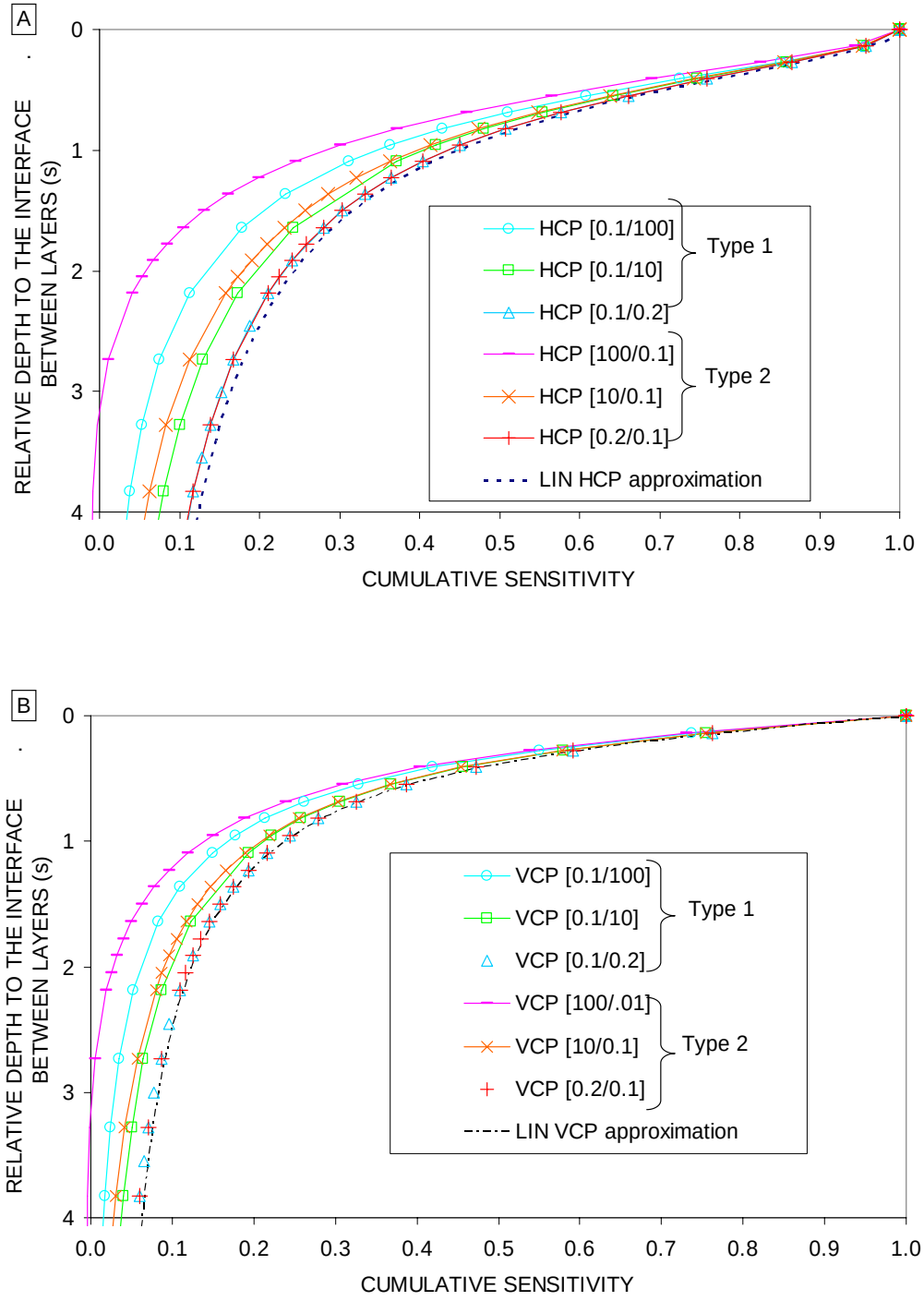

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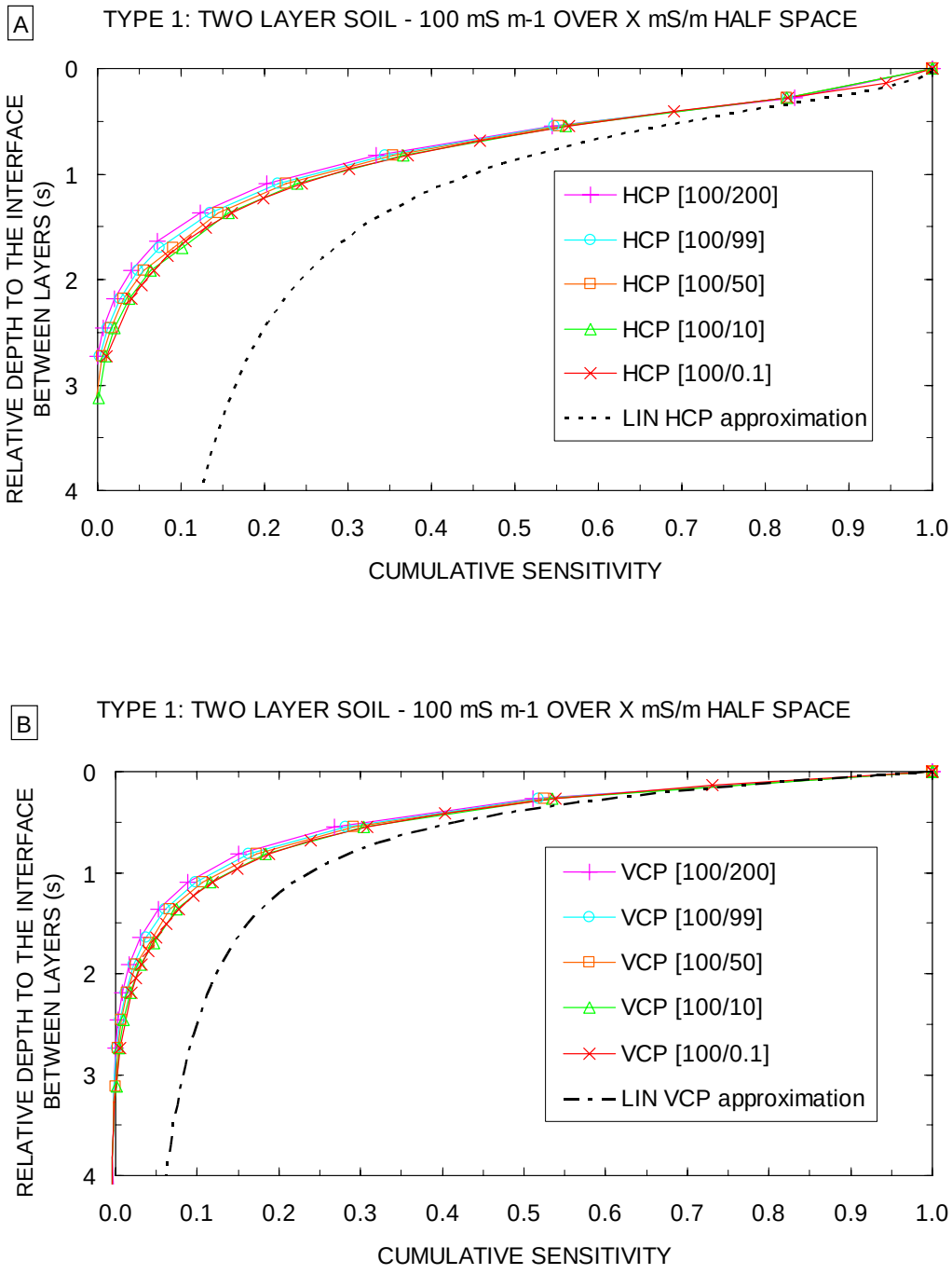

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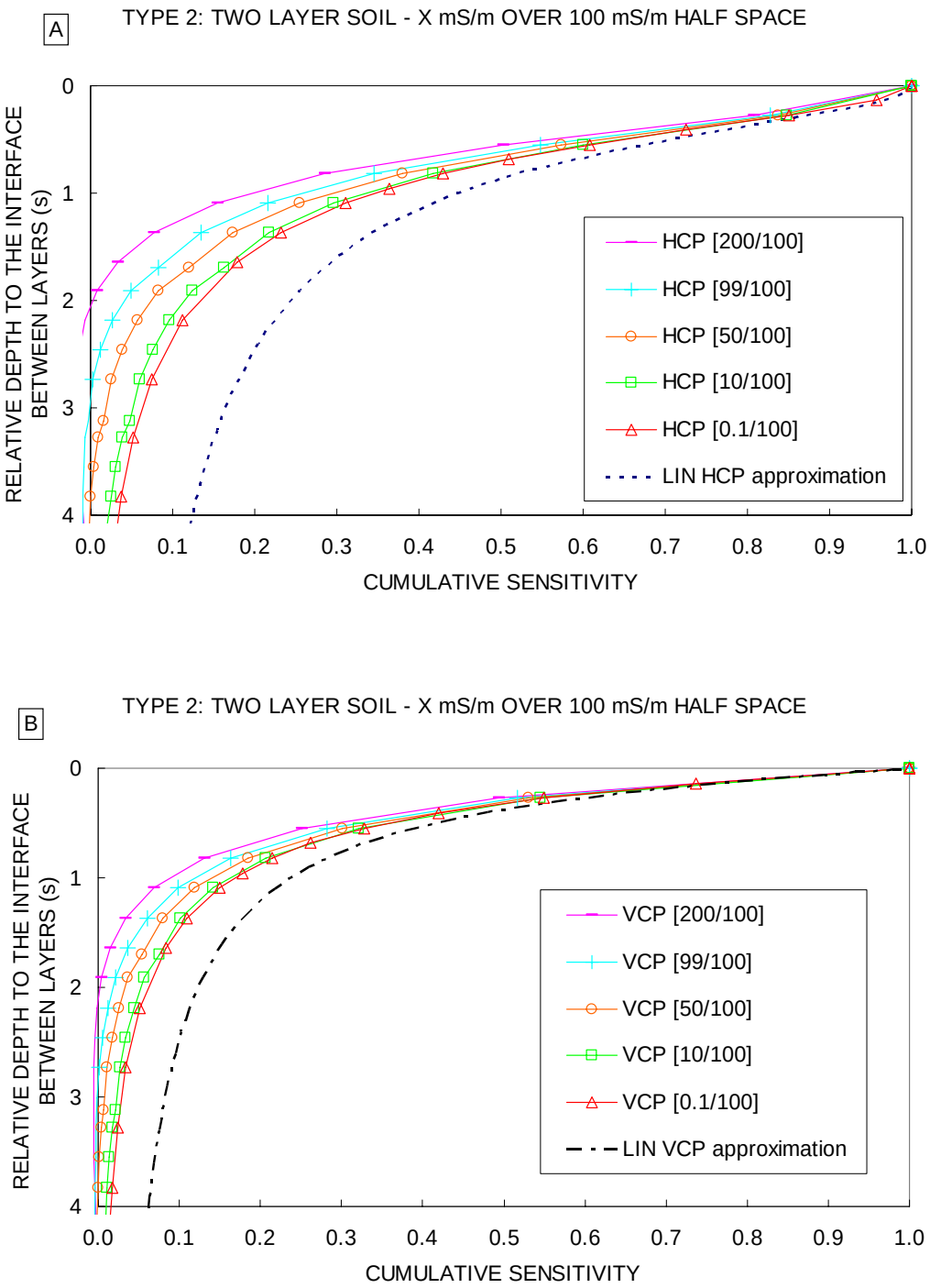
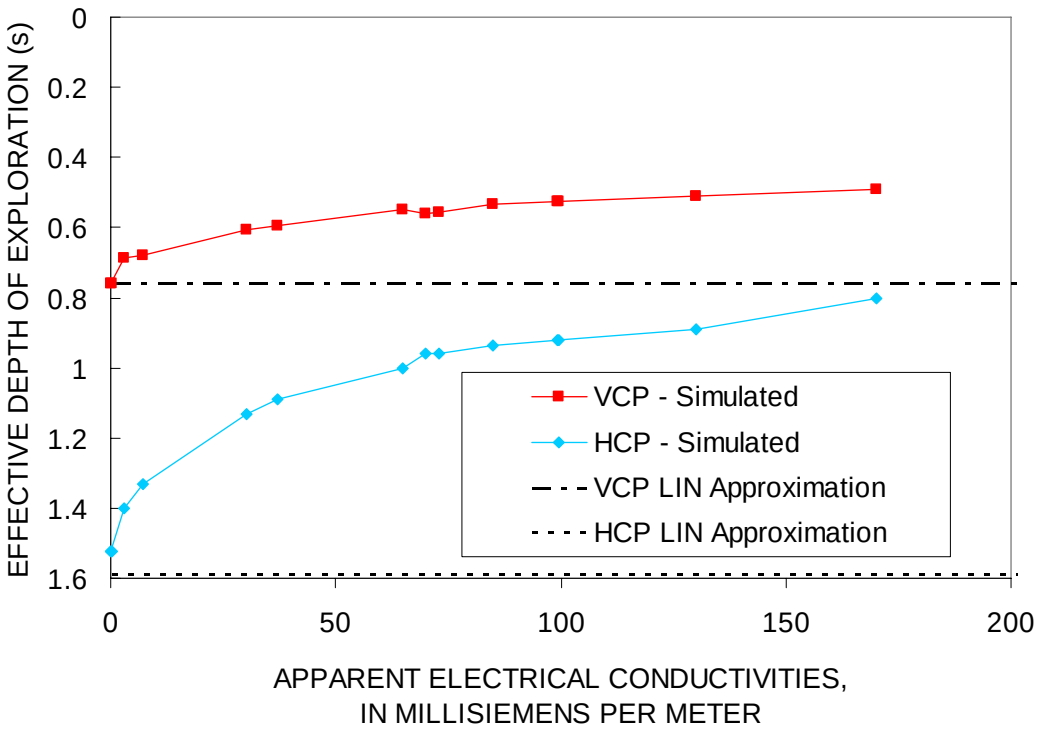


Figure 6. Soil model Type 2 cumulative sensitivity using the higher electrical conductivities sometimes detected in environmental investigations. Comparison of results of forward numerical model simulation results with the low-induction-number (LIN) approximation. (A) is horizontal (HCP) and (B) is vertical (VCP) coplanar coil orientation. Relative depth is the depth divided by the intercoil separation. The first number in brackets is the upper layer electrical conductivity, in mS m^{-1}, the second number is the lower layer electrical conductivity.



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